Math 10A with Professor Stankova
Quiz 9; Wednesday, 10/25/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** ____________________

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** If rate at which the area A changes is proportional to the radius r, then there exist constants C, D such that $\frac{dA}{dt} = Cr + D$.

Solution: There is no constant D.

2. **TRUE** False In order to show that the integral $0 \leq \int \frac{1}{f(x)} dx$ converges, it suffices to find a function $g(x)$ such that $f(x) \geq g(x)$ and show that $\int \frac{1}{g(x)} dx$ converges.

Solution: This is true because if $f(x) \geq g(x)$, then $\frac{1}{f(x)} \leq \frac{1}{g(x)}$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Suppose that $\frac{dy}{dx} = \sec(y)\sin(x)$. Find a solution such that $y(0) = \pi$.

Solution: We move things to different sides and get that

$$\frac{dy}{\sec(y)} = \cos(y)dy = \sin(x)dx.$$

Taking the integral of both sides gives

$$\sin(y) = -\cos(x) + C.$$

Now plugging in $y(0) = 0$, we have that

$$0 = \sin(\pi) = -\cos(0) + C = -1 + C$$

and so $C = 1$. Therefore, the equation is

$$\sin(y) = 1 - \cos(x).$$

(b) (3 points) Integrate $\int_2^\infty \frac{1}{(1-x)^2}dx$.

Solution: We have that

$$\int_2^\infty \frac{1}{(1-x)^2}dx = \lim_{t\to\infty}\int_2^t \frac{1}{(1-x)^2}dx = \lim_{t\to\infty} \frac{1}{1-x}\Big|_2^t = \lim_{t\to\infty} \frac{1}{1-t} - \frac{1}{1-2} = 0-(-1) = 1.$$

(c) (3 points) Does the integral $\int_2^\infty \frac{\sin^2(x)}{(1-x)^2+e^{-x}}dx$ converge? Hint: Use the previous part.

Solution: We know that $\sin^2(x) \leq 1$ and $(1-x)^2+e^{-x} \geq (1-x)^2$ so combining these two gives $\frac{\sin^2(x)}{(1-x)^2+e^{-x}} \leq \frac{1}{(1-x)^2}$. So, we have that

$$0 \leq \int_2^\infty \frac{\sin^2(x)}{(1-x)^2+e^{-x}}dx \leq \int_2^\infty \frac{1}{(1-x)^2}dx = 1,$$

and so the integral converges.